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Gravitational Dressing of D-Instantons

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Abstract

The non-perturbative corrections to the universal hypermultiplet moduli space metric in the type-IIA superstring compactification on a Calabi-Yau threefold are investigated in the presence of 4d, N=2 supergravity. These corrections come from multiple wrapping of the BPS (Euclidean) D2-branes around certain (BPS) Calabi-Yau 3-cycles, and they are known as the D-instantons. The exact universal hypermultiplet metric is governed by a quaternionic potential that satisfies the $SU(\infty)$ Toda equation. The mechanism is proposed, which elevates any four-dimensional hyper-Kähler metric with a rotational isometry to the quaternionic metric of the same dimension. A generic separable solution to the Toda equation appears to be related to the Eguchi-Hanson metric, whereas another solution originating from the Atiyah-Hitchin metric describes the gravitationally dressed (mixed) D-instantons.

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1 Introduction

Non-perturbative contributions in compactified M-theory/superstrings are believed to be crucial for solving the fundamental problems of vacuum degeneracy and supersymmetry breaking. Some instanton corrections to various physical quantities in the effective four-dimensional field theory, originating from the type-IIA superstring compactification on a Calabi-Yau (CY) threefold $\mathcal{Y}$, arise from the Euclidean (BPS) membranes and fivebranes wrapping CY cycles [1]. For instance, the corrections to the moduli space of the Universal Hypermultiplet (UH), present in any CY compactification of type-II strings [2], come from the D2-branes (membranes) wrapped about supersymmetric (BPS) 3-cycles $\mathcal{C}$ of $\mathcal{Y}$ and the NS5-branes (fivebranes) wrapped about the entire CY threefold [1]. The fivebranes give rise to the $e^{-1/g^2_{\rm string}}$ corrections to the UH effective action, whereas the wrapped D2-branes (known as the D-instantons [2]) result in the $e^{-1/g_{\rm string}}$ corrections, where $g_{\rm string}$ is the string coupling constant [4]. The D-instanton corrections to the UH moduli space metric were explicitly calculated in the hyper-Kähler limit where the 4d, N=2 supergravity decouples [5].

Without decoupling gravity, the UH moduli space metric is quaternionic [6]. The importance of the gravitational corrections to the UH quantum moduli space was emphasized, e.g., in refs. [7, 8, 9] where the perturbative gravitational correction [7] and the relevant instanton actions [8, 9] were calculated, but no D-instanton-corrected quaternionic metric was found. Being applied to the bosonic instanton background, broken supersymmetries generate fermionic zero-modes that have to be absorbed by extra terms in the effective field theory. These instanton-induced interactions are quartic in the fermionic fields and thus contribute to the curvature tensor of the supersymmetric Non-Linear Sigma-Model (NLSM) with the UH quaternionic metric [1]. To calculate the non-perturbative corrections to the quaternionic metric in this approach, one would have to integrate over the fermionic zero modes and compute the fluctuation determinants, which is apparently the hard problem [10].

In this Letter we fully exploit the special properties of the UH, whose quantum moduli space is highly constrained by quaternionic geometry and is independent upon local data about the underlying CY threefold $\mathcal{Y}$. We reduce the problem to an integrable non-linear differential equation on the quaternionic potential of the UH metric, whose solutions are capable to describe all D-instanton corrections in the presence of 4d, N=2 supergravity. We call it the gravitational dressing of hyper-Kähler D-instantons.

[2]The D-instantons may also be related to the type-IIA string world-sheet instantons [3].

The CY compactification *Ansatz* for the 10d metric of type-IIA strings is

$$ds^2_{10} = e^{-\phi/2}ds^2_{\rm CY} + e^{3\phi/2}g_{\mu\nu}dx^\mu dx^\nu \ , \tag{1}$$

where $g_{\mu\nu}$ is the 4d spacetime metric, $\mu,\nu = 0,1,2,3$, $ds^2_{\rm CY} = g_{i\bar{j}}dy^i d\bar{y}^{\bar{j}}$ is the internal CY metric, $i,j = 1,2,3$, and $\phi(x)$ is the 4d dilaton. Any CY threefold $\mathcal{Y}$ possesses the (1,1) Kähler form J and the holomorphic (3,0) form Ω by definition. We are going to consider the universal sector of the CY compactification (thus leaving aside the complex moduli of $\mathcal{Y}$), which is described by a single Universal Hypermultiplet (UH) comprising the dilaton ϕ, the axion D coming from dualizing the field strength $H_3 = dB_2$ of the NS-NS tensor field B_2 in 4d, and the complex scalar C coming from the RR three-form A_3, where $A_{ijk}(x,y) = \sqrt{2}C(x)\Omega_{ijk}(y)$. From the M-theory perspective, the dilaton expectation value represents the CY volume, whereas the expectation value of the RR scalar C is associated with the CY period $\int_{\mathcal{C}}\Omega$. The charge quantization of branes implies that the UH moduli space has to be periodic both in C and D [1, 8]. As long as the D-instanton contributions are concerned (i.e. no fivebranes), the axion merely appears via the Legendre transform, which implies the invariance with respect to constant shifts of D. The classical UH metric appears in place of the NLSM metric in the type-IIA supergravity action compactified on a rigid CY space with $g_{i\bar{j}} = \delta_{i\bar{j}}$ and $\Omega_{ijk} = \varepsilon_{ijk}$, after the Legendre transform [11],

$$ds^2_{\rm classical} = d\phi^2 + e^{2\phi}\left|dC\right|^2 + e^{4\phi}\left(dD + \tfrac{i}{2}\bar{C}\overleftrightarrow{d}C\right)^2 \ . \tag{2}$$

The classical UH moduli space is thus given by the symmetric space $SU(2,1)/U(2)$ whose standard (Bergmann) metric is equivalent to eq. (2) up to a Kähler gauge transformation and a field reparametrization [11]. The perturbative (one-loop) string corrections to the metric (2) originate from the R^4-term of M-theory action in 11d, while they are proportional to the Euler number $\chi = 2\left({\rm dim}H^{1,1}(\mathcal{Y}) - {\rm dim}H^{1,2}(\mathcal{Y})\right)$ of CY [7]. The one-loop corrected NLSM action of UH coupled to 4d gravity in the Einstein frame is given by [12]

$$\begin{aligned} S_{\rm perturbative} \ = \ & \int d^4x\sqrt{-g}\left\{\left(e^{-2\phi}+\hat{\chi}\right)\left(\tfrac{1}{2}R - \tfrac{1}{6}H^2_{\mu\nu\lambda}\right) + \frac{2e^{-4\phi}}{e^{-2\phi}+\hat{\chi}}(\partial_\mu\phi)^2\right\} \\ & - \int d^4x\sqrt{-g}\left[\partial_\mu C\partial^\mu\bar{C} + \tfrac{i}{2}H^\mu\left(C\partial_\mu\bar{C} - \bar{C}\partial_\mu C\right)\right] \ , \end{aligned} \tag{3}$$

where the 4d gravitational constant is $\kappa_4 = 1$, the conserved vector H_μ is the Hodge dual to $H_{\mu\nu\lambda}$ in 4d, and $\hat{\chi} \sim \chi$. The action (3) is invariant under the perturbative Peccei-Quinn symmetry $C \to C + const.$ This symmetry is going to be broken by the D-instantons to a discrete subgroup [1, 5]. The local NLSM metrics described by eqs. (2) and (3) are also equivalent up to a field reparametrization [1, 5]. In what follows we only consider the D-instantons (i.e. no fivebrane corrections).

2 Quantum hyper-Kähler UH metric

The D-instanton contributions to the classical UH metric (2) in *flat* 4d spacetime are given by hyper-Kähler deformations of the metric (2) under the preservation of the NLSM isometry associated with the translational invariance in the D-direction [5]. To calculate these deformations, it is natural to put the metric (2) into the canonical form (in terms of two potentials W and u, and a one-form Θ_1) [13, 15],

$$ds^2_{\rm K} \equiv g_{ab}d\phi^a d\phi^b = W^{-1}(dt+\Theta_1)^2 + W\left[e^u(dx^2+dy^2)+d\omega^2\right] \ , \tag{4}$$

valid for *any* Kähler metric g_{ab} in four real dimensions, $a,b=1,2,3,4$, with a Killing vector K^a that preserves the Kähler structure. We use the adapted coordinates $\phi^a=(t,x,y,\omega)$ where t is the coordinate along the trajectories of the Killing vector and (x,y,ω) are the coordinates in the space of trajectories, $W^{-1}=g_{ab}K^aK^b\neq 0$. The Kähler condition on the metric (4) implies a linear equation on Θ_1,

$$d\Theta_1 = W_x\, dy\wedge d\omega + W_y\, d\omega\wedge dx + (We^u)_\omega\, dx\wedge dy \ , \tag{5}$$

which, in turn, yields the following integrability condition on W [13]:

$$W_{xx}+W_{yy}+(We^u)_{\omega\omega}=0 \ . \tag{6}$$

The hyper-Kähler property means the existence of *three* independent Kähler structures $(J_k)_a{}^b$, $k=1,2,3$, which are covariantly constant, $\nabla_c (J_k)_a{}^b=0$, and obey the quaternionic algebra. Moreover, in four real dimensions, the hyper-Kähler property of the metric is equivalent to Anti-Self-Duality (ASD) of its Riemann curvature tensor [16]. As regards four-dimensional Kähler metrics, their Riemann ASD members are just Ricci-flat [17]. If the Killing vector K^a is triholomorphic (i.e. if the hyper-Kähler structure is inert under the isometry), one can further restrict the metric (4) by taking $u=0$. The Riemann ASD condition then amounts to a *linear* system [14],

$$\Delta W = 0 \quad \text{and} \quad \vec{\nabla}W + \vec{\nabla}\times\vec{\Theta} = 0 \ , \tag{7}$$

where Δ is the Laplace operator in three flat dimensions, $\Delta = 4g^2_{\rm string}\partial_z\bar{\partial}_{\bar z}+\partial^2_\omega$, and $z=g_{\rm string}(x+iy)$. The D-instanton potential W can now be thought of as the electrostatic potential for a collection of electric charges distributed in 3d, near the axis $z=0$ with unit density in ω. The unique regular (outside the positions of charges) solution to this problem in the limit $g_{\rm string}\to 0$, while keeping $|z|/g_{\rm string}$ finite, reads [18]

$$W = \frac{1}{4\pi}\log\left(\frac{\mu^2}{z\bar z}\right) + \sum_{m=1}^{\infty}\frac{\cos(2\pi m\omega)}{\pi}K_0\left(\frac{2\pi\,|mz|}{g_{\rm string}}\right) \ , \tag{8}$$

where K_0 is the modified Bessel function. The solution (8) can be trusted for large $|z|$, where it amounts to the infinite D-instanton/anti-instanton sum [5],

$$
\begin{aligned}
W \; = \; & \frac{1}{4\pi}\log\left(\frac{\mu^2}{z\bar{z}}\right)+\sum_{m=1}^{\infty}\exp\left(-\frac{2\pi\,|mz|}{g_{\rm string}}\right)\cos(m\omega) \\
& \times\sum_{n=0}^{\infty}\frac{\Gamma(n+\frac{1}{2})}{\sqrt{\pi}n!\Gamma(-n+\frac{1}{2})}\left(\frac{g_{\rm string}}{4\pi\,|mz|}\right)^{n+\frac{1}{2}} \quad .
\end{aligned}
\tag{9}
$$

If we merely required the vanishing *scalar* curvature of the Kähler metric (4) or a non-triholomorphic isometry of the hyper-Kähler metric, we would get the non-linear equation [13, 14]

$$
u_{xx}+u_{yy}+(e^u)_{\omega\omega}=0 \ , \tag{10}
$$

which is known as the $SU(\infty)$ Toda field equation [19, 15] since it appears in the large-N limit of the standard two-dimensional Toda system for $SU(N)$.

3 Quantum quaternionic UH metric

A quaternionic manifold admits three independent *almost* complex structures $(\tilde{J}_k)_a{}^b$, which are, however, *not* covariantly constant but satisfy $\nabla_a(\tilde{J}_k)_b{}^c=(T_a)_k{}^n(\tilde{J}_n)_b{}^c$, where $(T_a)_k{}^n$ is the NLSM torsion [16]. This torsion is induced by 4d gravity because the quaternionic condition on the hypermultiplet NLSM metric is the direct consequence of *local* N=2 supersymmetry in four spacetime dimensions [6]. As regards four-dimensional quaternionic manifolds (relevant for UH), they all have *Einstein-Weyl* geometry of the *negative* scalar curvature [16, 6], i.e.

$$
W^-_{abcd}=0 \ , \qquad R_{ab}=\frac{\Lambda}{2}g_{ab} \ , \tag{12}
$$

where W_{abcd} is the Weyl tensor and R_{ab} is the Ricci tensor for the metric g_{ab}. The overall coupling constant of the 4d NLSM has the same dimension as κ^2, while in the N=2 locally supersymmetric NLSM these coupling constants are proportional to each other with the dimensionless coefficient $\Lambda<0$ [6].

Since the quaternionic and hyper-Kähler conditions are not compatible, the canonical form (4) should be revised. [3] It is remarkable, however, that the exact quaternionic metric is governed by *the same* three-dimensional Toda equation (10). For example, when using the general *Ansatz* [20]

$$
ds^2_{\rm Q}=\frac{P}{\omega^2}\left[e^u(dx^2+dy^2)+d\omega^2\right]+\frac{1}{P\omega^2}(dt+\Theta_1)^2 \tag{13}
$$

[3] A generic Einstein-Weyl manifold does not have a Kähler structure.

for a quaternionic four-dimensional metric with an isometry, it is straightforward (albeit tedious) to prove that the restrictions (12) on the metric (13) *precisely* yield eq. (10) on the potential $u = u(x, y, \omega)$, while P is given by [20]

$$P = \frac{1}{2\Lambda}\left(\omega u_\omega - 2\right) , \tag{14}$$

and Θ_1 obeys the linear equation

$$d\Theta_1 = -P_x\, dy \wedge d\omega - P_y\, d\omega \wedge dx - e^u(P_\omega + \tfrac{2}{\omega}P + \tfrac{2\Lambda}{\omega}P^2)dx \wedge dy \ . \tag{15}$$

The limit $\Lambda \to 0$, where 4d gravity decouples, should be taken with care. After rescaling $u \to \Lambda u$ in eq. (10) we get

$$u_{xx} + u_{yy} + \frac{1}{\Lambda}(e^{\Lambda u})_{\omega\omega} = 0 \ , \tag{16}$$

which yields the 3d Laplace (linear!) equation when $\Lambda \to 0$ indeed. We conclude that the non-linear eq. (10) substitutes the linear eq. (7) for the UH metric potential in the presence of 4d, N=2 supergravity.

4 Exact quaternionic solutions

In terms of the complex coordinate $\zeta = x + iy$, the 3d Toda equation (10) takes the form

$$4u_{\zeta\bar{\zeta}} + (e^u)_{\omega\omega} = 0 \ . \tag{17}$$

It is not difficult to check that this equation is invariant under *holomorphic* transformations of ζ,

$$\zeta \to \hat{\zeta} = f(\zeta) \ , \tag{18}$$

with arbitrary function $f(\zeta)$, provided that it is accompanied by the shift of the Toda potential,

$$u \to \hat{u} = u - \log(f') - \log(\bar{f}') \ , \tag{19}$$

where the prime means differentiation with respect to ζ or $\bar{\zeta}$, respectively. The transformations (18) can be interpreted as the residual diffeomorphisms in the NLSM target space of the universal hypermultiplet, which keep invariant the quaternionic Ansatz (13) under the compensating 'Toda gauge transformations' (19).

To make contact with the results of the preceeding sections, let's first search for *separable* exact solutions to the Toda equation, having the form

$$u(\zeta, \bar{\zeta}, \omega) = F(\zeta, \bar{\zeta}) + G(\omega) \ . \tag{20}$$

Equation (16) then reduces to two separate equations,

$$F_{\zeta\bar{\zeta}} + \frac{c^2}{2} e^F = 0 \tag{21}$$

and

$$\partial_\omega^2 e^G = 2c^2 \ , \tag{22}$$

where c^2 is a separation constant. After taking into account the positivity of e^G, the general solution to eq. (22) reads

$$e^G = c^2(\omega^2 + 2\omega b \cos\alpha + b^2) \ , \tag{23}$$

where b and α are arbitrary real integration constants.

Equation (21) is the 2d *Liouville* equation that is well known in 2d quantum gravity [21]. Its general solution reads

$$e^F = \frac{4\,|f'|^2}{(1 + c^2\,|f|^2)^2} \tag{24}$$

in terms of arbitrary holomorphic function $f(\zeta)$. The ambiguity associated with this function is, however, precisely compensated by the Toda gauge transformation (19), so that we have the right to choose $f(\zeta) = \zeta$ in eq. (24). This yields the following regular exact solution to the 3d Toda equation:

$$e^u = \frac{4c^2(\omega^2 + 2\omega b \cos\alpha + b^2)}{(1 + c^2\,|\zeta|^2)^2} \ . \tag{25}$$

It is obvious now that the konstant c^2 is positive indeed. It also follows from eqs. (13), (14) and (25) that the separable exact solution to the quaternionic UH metric possesses the rigid $U(1)$ duality symmetry with respect to the phase rotations $\zeta \to e^{i\theta}\zeta$ of the complex RR-field ζ.

Though the quaternionic NLSM metric defined by eqs. (13), (14), (15) and (25) is apparently different from the classical UH metric (2), these metrics are nevertheless equivalent in the classical region of the UH moduli space where all quantum corrections are suppressed. The classical approximation corresponds to the conformal limit $\omega \to \infty$ and $|\zeta| \to \infty$, while keeping the ratio $|\zeta|^2/\omega$ finite. Then one easily finds that $P \to -\Lambda^{-1} = const. > 0$, whereas the metric (13) takes the form

$$ds^2 = \frac{1}{\lambda^2}\left(|dC|^2 + d\lambda^2\right) + \frac{1}{\lambda^4}(dD + \Theta)^2 \ , \tag{26}$$

in terms of the new variables $C = 1/\zeta$ and $\lambda^2 = \omega$, after a few rescalings. The metric (26) reduces to that of eq. (2) when using $\lambda^{-2} = e^{2\phi}$. Another interesting limit is $\omega \to 0$ and $|\zeta| \to \infty$, where one gets a conformally flat metric (AdS_4).

Having established that the hyper-Kähler and quaternionic metrics under consideration are governed by the same Toda equation (sects. 2 and 3), the new mechanism [4] of generating the quaternionic metrics from known hyper-Kähler metrics in the same (four) dimensions arises: first, one deduces a solution to the Toda equation (10) from a given four-dimensional hyper-Kähler metric having a non-triholomorphic or rotational isometry, by rewriting it to the form (4), and then one inserts the obtained exact solution into the quaternionic Ansatz (13) to deduce the corresponding quaternionic metric with the same isometry. Being applied to the D-instantons, this mechanism results in their gravitational dressing with respect to 4d, N=2 supergravity.

The $SU(\infty)$ Toda equation is known to be notoriously difficult to solve, while a very few its exact solutons are known. Nevertheless, the proposed connection to the hyper-Kähler metrics can be used as the powerful vehicle for generating exact solutions to eq. (10). It is worth mentioning that eq. (6) follows from eq. (10) after a substitution

$$W = \partial_\omega u \ , \tag{27}$$

while eq. (5) is solved by

$$\Theta_1 = \mp \partial_y u(dx) \pm \partial_x u(dy) \ . \tag{28}$$

This is known as the Toda frame for a hyper-Kähler metric [14, 15]. It is not difficult to verify that the separable solution (25) is generated from the Eguchi-Hanson (hyper-Kähler) metric along these lines [15]. A highly non-trivial solution to the Toda equation (10) follows from the Atiyah-Hitchin (hyper-Kähler) metric [17]. The transform to the Toda frame reads [23]

$$y + \mathrm{i}x = K(k)\sqrt{1+k'^2\sinh^2\nu}\left(\cos\vartheta + \frac{\tanh\nu}{K(k)}\int_0^{\pi/2}\mathrm{d}\gamma\,\frac{\sqrt{1-k^2\sin^2\gamma}}{1-k^2\tanh^2\nu\sin^2\gamma}\right)$$

$$\omega = \frac{1}{8}K^2(k)\left(k^2\sin^2\vartheta + k'^2(1+\sin^2\vartheta\sin^2\psi) - \frac{2E(k)}{K(k)}\right) , \tag{29}$$

where $(\vartheta,\psi,\varphi;k)$ are the new coordinates (in four dimensions). The parameter k plays the role of modulus here, $0<k<1$, while $k' = \sqrt{1-k^2}$ is called the complementary modulus. The remaining definitions are

$$\nu \equiv \log\left(\tan\frac{\vartheta}{2}\right) + i\psi \ , \quad \tau = 2\left(\varphi + \arg(1+k'^2\sinh^2\nu)\right) \ , \tag{30}$$

[4] The different mechanism, which associates the quaternionic metric in $4(n+1)$ real dimensions to a given (special) hyper-Kähler metric in $4n$ real dimensions, was proposed in ref. [22].

in terms of the standard complete elliptic integrals of the first and second kind, $K(k)$ and $E(k)$, respectively [15]. The associated solution to the Toda equation (10) reads [24, 15]

$$\mathrm{e}^{u} = \frac{1}{16} K^{2}(k) \sin^{2}\vartheta \left| 1 + k'^{2} \sinh^{2}\nu \right| \ . \tag{31}$$

The physical significance of the related quaternionic metric solution becomes clear in the perturbative region $k \to 1$, where [25]

$$k' \propto e^{-S_{\text{inst.}}} \ , \quad \text{and} \quad S_{\text{inst.}} \to +\infty \ . \tag{32}$$

In this limit the Atiyah-Hitchin metric is exponentially close to the Taub-NUT metric [17], while the exponentially small corrections can be interpreted as the (mixed) D-instantons. The D-instanton action $S_{\text{inst.}}$ is essentially given by the volume of the supersymmetric three-cycle $\mathcal{C}$. The same exact solution also describes the hypermultiplet moduli space metric in the 3d, N=4 supersymmetric Yang-Mills theory with the $SU(2)$ gauge group, which was obtained via the c-map in ref. [26].

More general regular hyper-Kähler four-manifolds with a rotational isometry are known [27], albeit in the rather implicit form (as algebraic curves), which does not allow us to explicitly transform their metrics to the Toda frame. It is also known that the Toda equation (10) can be reduced in a highly non-trivial way to the Painlevé equations [28]. If the non-abelian $SU(2)$ isometry is preserved, the UH quaternionic metric can be interpreted as the N=2 supersymmetric gradient flow (domain wall), whose explicit form is known in terms of theta functions [29].

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